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MIXING IN A SYSTEM OF PLANE COCURRENT JETS OF AN INCOMPRESSIBLE LIQUID OVER THE MAIN PORTION

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We consider mixing in a periodic system of plane subsonic jets of an incompressible liquid for laminar and turbulent flows. Asymptotic expressions for the change in the velocity defect are obtained. A comparison with a numerical calculation and with existing experimental data is carried out. We show that under the conditions considered the accuracy of the expressions derived is rather high and they can be used for evaluating the mixing velocity over the main portion of mixing.

Systems of plane parallel jets are met with in technological devices, combustion chambers, gas lasers, etc. Moreover, such a system is the simplest model for complex three-dimensional systems of jets widely used in technology. In view of this, determination of the mixing velocity in a system of plane jets is of practical interest. For this purpose use is frequently made of methods developed for mixing a single jet; however, use of these methods to calculate mixing of a system of jets leads to incorrect results, since the regularities of the process are of a different nature in this case.

A theoretical solution of this problem for turbulent gas jets was suggested in a number of works (e.g., [1-5]); the authors of these works used mainly numerical methods. Good agreement with experiment was obtained on the assumption that the turbulent viscosity was constant over the main portion of mixing; however, the general regularities of the change in the velocity defect and other quantities remained unclarified.

Below we consider an infinite periodic system of plane subsonic cocurrent jets of an incompressible liquid under conditions where it is possible to use two-dimensional equations in the narrow-channel approximation. In the initial cross section the profiles of the longitudinal velocity, temperature, and concentration have a stepwise character. The computational domain is bounded by the axes of two adjacent jets. Boundary conditions of symmetry are set on these axes. We will begin our consideration with the simplest case of laminar flow. We write the equations of continuity and motion for plane jets in the narrow channel approximation:

$$\partial u/\partial x + \partial v/\partial y = 0 , \tag{1}$$

$$u \, \partial u/\partial x + v \, \partial u/\partial y = \nu \, \partial^2 u/\partial y^2 - (1/\rho) \, dP/dx \tag{2}$$

with boundary conditions on the axes of the jets:

$$y = 0, \pm L, \pm 2L , \dots : \ \partial u/\partial y = 0 , \ \ v = 0 .$$

By virtue of the system periodicity, it is sufficient to consider the band $0 \le y \le L$. Initial conditions are assigned in the cross section $x = 0$:

$$u(0, y) = u_1 \ \text{ for } \ 0 \le y < l_1 ; \ \ u(0, y) = u_2 \ \text{ for } \ l_1 \le y < L .$$

At large values of the longitudinal coordinate the transverse velocity component tends to zero, and the longitudinal velocity component tends to a certain constant quantity u_∞ that can be determined from the condition of flow-rate conservation: $u_1 l_1 + u_2 l_2 = u_\infty L$, where $l_2 = L - l_1$.

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Now we reduce the equations to dimensionless form, taking the quantity L as the scale of the transverse coordinate, the quantity $\lambda = \mathrm{Re}L/\pi^2$ as the scale of the longitudinal coordinate, and the quantity u_∞ as the velocity scale. The Reynolds number is determined by the parameters of the completely mixed flow: $\mathrm{Re} = u_\infty L/\nu$.

We expand the longitudinal and transverse velocity components into a Fourier series in the coordinate $\bar{y}$. With allowance for continuity equation (1) and the boundary conditions we have

$$\bar{u} = 1 + \sum_{j=1}^{\infty} a_j(\bar{x}) \cos(\pi j \bar{y}), \tag{3}$$

$$\bar{v} = -\sum_{j=1}^{\infty} (L/\pi j \lambda)\, a_j'(\bar{x}) \sin(\pi j \bar{y}). \tag{4}$$

The prime denotes differentiation with respect to $\bar{x}$.

From the integral law of momentum conservation we obtain an expression for the pressure gradient:

$$d\bar{P}/d\bar{x} = -\sum_{j=1}^{\infty} a_j a_j' \tag{5}$$

where

$$\bar{P} = P/\rho_\infty u_\infty^2. \tag{6}$$

Substituting expansions (3)-(5) into Eq. (2), we derive a system of equations for the coefficients a_j:

$$a_j' + j^2 a_j = -F_j(\bar{x}), \quad j = 1, 2, 3, \dots, \tag{7}$$

in which all the nonlinear terms are grouped in F_j.

Formally, the solution of (7) with allowance for the initial conditions can be written in the form

$$a_j(\bar{x}) = \exp(-j^2\bar{x}) \left\{ a_{j0} - \int_0^{\bar{x}} F_j(\bar{x}) \exp(j^2\bar{x})\, d\bar{x} \right\}. \tag{8}$$

The coefficients a_{j0} are determined using the expansion of the velocity profile into a Fourier series in the initial cross section, and under the initial conditions prescribed above they are equal to

$$a_{j0} = (2/\pi j)\, \Delta\bar{u}_0 \sin(\pi j l_1/L), \tag{9}$$

where $\Delta\bar{u}_0 = (u_1 - u_2)/u_\infty$ is the dimensionless initial difference in the velocities of the jets.

In the particular case where the transverse dimensions of the adjacent jets are equal, we have

$$a_{j0} = \begin{cases} (-1)^n\, 2\Delta\bar{u}_0/\pi j, & j = 2n+1, \\ 0, & j = 2n. \end{cases} \tag{10}$$

Solution (8) is sought by means of successive approximations using, as the zero approximation, the distribution of the quantities at an infinite distance from the initial cross section (i.e., in the completely mixed flow, when $a_j^{(0)} = 0$). We substitute the expressions for $a_j(\bar{x})$ obtained at the next step into $F_j(\bar{x})$, etc.

At the first step of the procedure we obtain

$$a_j^{(1)} = a_{j0} \exp(-j^2\bar{x}).$$

Thus, the first approximation coincides with the solution of the linearized equation $\partial\bar{u}/\partial\bar{x} = (1/\pi^2)\partial^2\bar{u}/\partial\bar{y}^2$.

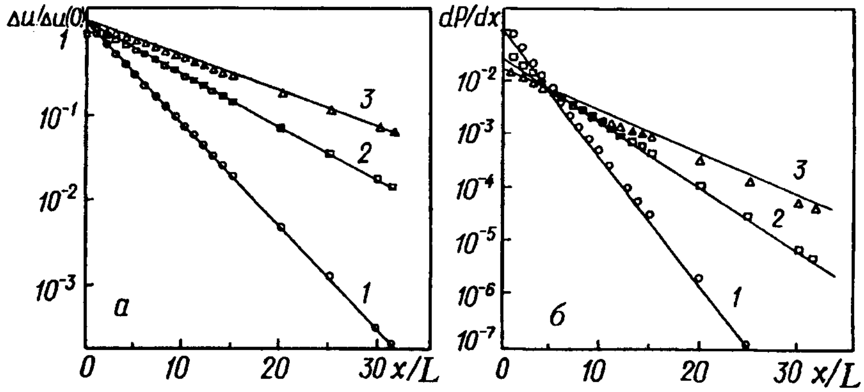

Fig. 1. Change in the velocity defect (a) and the dimensionless pressure gradient (b) for laminar mixing and Re = 35 (1), 70 (2), 105 (3); curves, formulas (11) (a) and (13) (b); points, numerical calculation.

The second approximation for the coefficients a_j has the form of a series of decreasing exponents. For the distant region of mixing it might be expected that already the first approximations describe rather accurately the solution of system (1)-(2).

For the difference in the velocities on the axes of the adjacent jets $\Delta u = u(x, 0) - u(x, L)$ we have the following expression:

$$\Delta\bar{u} = 2\,(a_1(\bar{x}) + a_3(\bar{x}) + a_5(\bar{x}) + \ldots)\,.$$

At large distances from the initial cross section the change in $\Delta\bar{u}(\bar{x})$ will be determined by the most slowly decreasing exponent, namely, $\exp(-\bar{x})$, which enters the coefficient $a_1(\bar{x})$ in the first harmonic. Thus, already the first approximation (or the solution of the linearized equation) gives the asymptotically correct behavior of $\Delta\bar{u}(\bar{x})$ for $x \to \infty$:

$$\Delta u/\Delta u_0 \approx C_u \exp(-x/\lambda)\,. \tag{11}$$

We determine the coefficient C_u from the second approximation:

$$C_u = (2A_1/\Delta\bar{u}_0) = (4/\pi) \times$$

$$\times \left\{\sin(\pi l_1/L) + (\Delta\bar{u}_0/2\pi) \sum_{i=1}^{\infty} \left(\frac{2i+1}{i\,(i+1)}\right)^2 \sin(i\pi l_1/L)\sin((i+1)\,\pi l_1/L)\right\}. \tag{12}$$

In the case $l_1 = l_2 = L/2$, $A_1 = a_{10}$ (since all the products $a_{i0}a_{i+1,0}$ are equal to zero), i.e., the preexponential factors in Eq. (11) obtained in the first and second approximations also coincide. As the number i increases, the terms of the series in Eq. (12) decrease as $1/i^2$. This makes it possible to confine oneself in the calculation to a finite number of terms. To find the sum of the series with an accuracy of 10%, it is sufficient to take into account 4 to 10 terms depending on the ratio l_1/L.

From expression (11) the meaning of the quantity λ becomes clear: it is the characteristic length of mixing along which the velocity defect decreases by a factor of e. This quantity is determined by the integral characteristics of the flow; the preexponential factor C_u depends on the ratio of the dimensions and the initial velocities of the jets: $C_u = C_u(l_1/L, \Delta\bar{u}_0)$. When the transverse dimensions of the jets are equal ($l_1/L = 1/2$), the dependence on $\Delta\bar{u}_0$ drops out and C_u becomes constant: $C_u = 4/\pi$, i.e., the mixing of the flows over the main portion in this case is determined by just one quantity, namely, the Reynolds number calculated from the parameters of the mixed flow.

The expression for the dimensionless pressure gradient corresponding to Eq. (11) has the form

$$d\bar{P}/dx \approx (A_1/\lambda)^2 \exp(-2x/\lambda). \tag{13}$$

Results of numerical solution of Eqs. (1) and (2) confirm the suitability of the expressions obtained. Figure 1a illustrates the change in the velocity defect obtained in the numerical calculation and from formula (11) for $l_1 = l_2 = L/2$, $\Delta\bar{u}_0 = 0.667$ (or $u_1/u_2 = 2$). A graph of the change in the dimensionless pressure gradient under the same conditions is given in Fig. 1b. It is seen that the accuracy of approximate asymptotic solutions (11) and (13) is rather high already at the beginning of the main portion of mixing.

From the results of the numerical calculations one can see that the expressions obtained remain valid in a wide range of change of the difference in the velocities of the flows and the ratio of their half-heights, at least up to $u_1/u_2 = 5$ and $l_1/l_2 = 3$.

Now we will consider turbulent mixing. For this, we use the gradient hypothesis for turbulent stresses. It is assumed that the turbulent viscosity is much greater than the intrinsic gas viscosity.

Instead of Eq. (2) we will have the expression (the special symbols for averaged quantities are omitted)

$$u\,\partial u/\partial x + v\,\partial u/\partial y = \partial/\partial y\,(\nu_t\,\partial u/\partial y) - (1/\rho)\,dP/dx. \tag{14}$$

We assume that the turbulent viscosity ν_t over the portion of interest changes but slightly over the cross section. The validity of this assumption will be shown below. Then, having defined the scale of the longitudinal coordinate as $\lambda = \mathrm{Re}L/\pi^2$, $\mathrm{Re} = u_\infty L/\nu_{t\infty}$, we obtain, similarly to the previous case, an equation for the Fourier coefficients:

$$da_j/d\bar{x} + j^2\,\bar{\nu}_t(\bar{x})\,a_j = -F_j(\bar{x}), \quad j = 1, 2, 3, \ldots \tag{15}$$

In order to integrate this equation, it is necessary to determine the specific form of the dependence $\bar{\nu}_t(\bar{x})$. For this purpose we will consider two well-known models of turbulence.

a) *The "new" Prandtl model*

$$\nu_t = \kappa l \Delta u, \tag{16}$$

where l is the scale, which is usually taken to be the jet width; here we take $l = L$. Within the framework of the present model the turbulent viscosity is constant over the cross section. The use, in Eq. (16), of the difference in the velocities on the axes of adjacent flows, which decreases downward along the flow, leads to the following asymptotic behavior of the velocity defect in the limit as $\bar{x} \to \infty$:

$$\Delta u/u_\infty \sim \bar{x}^{-1}.$$

Numerical calculations carried out using the "new" Prandtl model give the same result. However the experiments conducted in [3-5] indicate an exponential drop in Δu over the main portion. Possibly, this discrepancy is associated with the fact that in reality the scale l is not constant and it increases in such a way that it compensates for the decrease in Δu.

In [3, 4], in calculations of turbulent mixing using the Prandtl model it was assumed that the turbulent viscosity over the main portion of mixing was constant and equal to $\kappa L \Delta u_0$. Use of this combination gives for the velocity defect an expression that is completely analogous to that obtained for laminar mixing:

$$\Delta u/\Delta u_0 = C_u \exp(-x/\lambda).$$

The coefficient C_u is prescribed by formula (12). Taking into account the definition of turbulent viscosity within the framework of the model adopted, we have $\mathrm{Re}_t = u_\infty/\Delta u_0 \kappa$. Thus, according to the given model, the velocity defect decreases over the main portion exponentially and more rapidly, the greater the initial difference in the velocities of the jets. It should be noted that the coefficient C_u before the exponent also depends on the initial difference in the velocities.

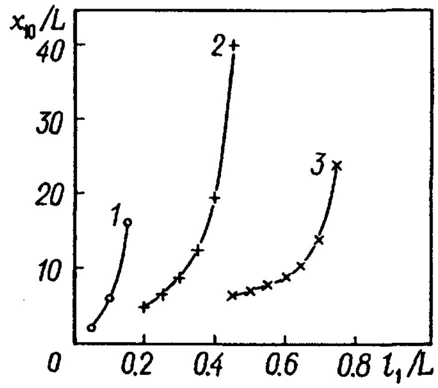

Fig. 2. X_{10}/L versus the ratio of the half-heights of adjacent jets for a ratio of the flow rates of 0.25 (1), 1 (2), 4 (3).

Now we consider the question of the influence of the initial parameters on the process of mixing. We assume that the miscible jets differ in composition; the density is considered to be constant. The equation for the concentration of a mixture component has the form

$$\bar{u}\,\partial Y_k/\partial \bar{x} + \bar{v}\,\partial Y_k/\partial \bar{y} = (1/\mathrm{Sc}_t\pi^2)\,\partial^2 Y_k/\partial \bar{y}^2$$

with the boundary conditions:

$$\bar{y} = 0{,}1: \quad \partial Y_k/\partial \bar{y} = 0\,.$$

Here Sc_t is the turbulent analog of the Schmidt number.

In the same way as was done for the velocity, it is possible to obtain the following expression for the change in the concentration defect over the main portion:

$$\Delta Y_k/\Delta Y_{k0} = C_k \exp\left(-\,x/\lambda_c\right), \tag{17}$$

where $\lambda_c = \mathrm{Re}_t\mathrm{Sc}_t L/\pi^2 = \mathrm{Sc}_t L/k\pi^2\Delta\bar{u}_0$ and $C_k = C_u$.

Figure 2 presents the dependence of the length x_{10}/L, over which the initial difference in the concentrations decreases by a factor of 10, on the ratio between the half-heights of the jets for different ratios of the flow rates. For each curve the flow rates were assumed to be fixed and $u_1 \geq u_2$. From this it is clear that an increase in l_1/L is equivalent to a decrease in $\Delta\bar{u}_0$. Since within the scope of the given model the turbulent viscosity is completely determined by the difference in the jet velocities, with decreasing $\Delta\bar{u}_0$ the mixing is sharply retarded.

We note that the expression used for the turbulent viscosity ignores the initial turbulence of the jets. In view of this, we will consider a more complex model.

b) The $k-\varepsilon$ model

$$u\,\partial k/\partial x + v\,\partial k/\partial y = \partial/\partial y\,(\nu_t/\sigma_k\,\partial k/\partial y) + \nu_t\,(\partial u/\partial y)^2 - \varepsilon\,,$$

$$u\,\partial \varepsilon/\partial x + v\,\partial \varepsilon/\partial y = \partial/\partial y\,(\nu_t/\sigma_\varepsilon\,\partial \varepsilon/\partial y) +$$

$$+\,C_1\,(\varepsilon/k)\,\nu_t\,(\partial u/\partial y)^2 - C_2\,\varepsilon^2/k\,,$$

$$\nu_t = C_\mu\,k^2/\varepsilon\,.$$

The boundary conditions are:

$$y = 0{,}L: \quad \partial k/\partial y = \partial \varepsilon/\partial y = 0\,.$$

To evaluate the behavior of the turbulent viscosity over the main portion, we will avail ourselves of the following considerations. At large distances from the initial cross section the changes in the parameters across the flow become insignificant. Consequently, to determine the asymptotic behavior of the turbulence parameters, we must solve the equations

$$u_\infty dk/dx = -\varepsilon\,;\quad u_\infty d\varepsilon/dx = -C_2\varepsilon^2/k\,.$$

We integrate this system of equations and, as the final result, obtain an expression for the turbulent viscosity:

$$\nu_t/\nu_{t0} = (1 + (C_2 - 1)\,(\varepsilon_0/\,k_0 u_\infty)\,x)^{-(2-C_2)/(C_2-1)}\,. \tag{18}$$

For the typical value of $C_2 = 1.92$ the exponent is equal to -0.087, i.e., the turbulent viscosity changes but slightly, and over a not very large interval it can be considered to be constant. To evaluate this interval, we take the following relation between ε_0 and k_0: $\varepsilon_0 = 0.3k_0^{3/2}/L$. Then, the dimensionless length along which the turbulent viscosity changes by 5% is expressed as

$$\overline{X}_{5\%} = 2.9\,u_\infty/k_0^{1/2}\,,$$

i.e., the lower the initial intensity of turbulence, the larger the distances at which the assumption of constancy of ν_t is valid. Thus, when $k_0/u_\infty^2 = 0.01$, $\overline{X}_{5\%} = 29$, while when $k_0/u_\infty^2 = 0.1$, $\overline{X}_{5\%} = 9$. If we take into consideration that with an increase in the initial level of turbulence, mixing occurs more rapidly, it can be concluded that the assumption of constancy of the turbulent viscosity over the main portion does not lead to substantial errors in determining the length of mixing.

The same conclusion follows from numerical calculations carried out using the $k-\varepsilon$ turbulence model; from these calculations one can see that at a sufficient distance from the initial portion of mixing the turbulent viscosity changes comparatively slightly over both the transverse and longitudinal coordinates and that it can be taken to be constant. Assuming the turbulent viscosity to be constant over the main portion, we obtain an expression similar to formula (11) for laminar mixing:

$$\Delta u/\Delta u_0 = C_u \exp{(-\,x/\lambda)}\,. \tag{19}$$

Here $\lambda = \mathrm{Re}_t L/\pi^2$, $\mathrm{Re}_t = Lu_\infty/\nu_{t.ef}$, and $\nu_{t.ef}$ is a certain effective value of the turbulent viscosity for the main portion. This value should be determined by the initial intensity of the jet turbulence and the generation and dissipation of turbulence in the layer of mixing.

To construct an appropriate approximation of $\nu_{t.ef}$ as a function of k_0 and Δu_0, we will avail ourselves of the following considerations. We write averaged equations for k and ε over a cross section and assume that all the turbulence parameters (k, ε, ν_t) change only slightly over the jet cross section:

$$dk/dt = \nu_t\,\langle\,(\partial u/\partial y)^2\rangle - \varepsilon\,, \tag{20}$$

$$d\varepsilon/dt = C_1\nu_t\,(\varepsilon/k)\,\langle(\partial u/\partial y)^2\rangle - C_2\varepsilon^2/k\,. \tag{21}$$

Here, the angular brackets denote averaging over the jet cross section.

Now we introduce the auxiliary function φ defined by the equality

$$\varepsilon/k = \varphi\,/(C_2 - 1)\,\varphi\,, \tag{22}$$

where the prime denotes differentiation with respect to t. Using Eqs. (20) and (21), for φ we obtain the expression

$$\varphi'' = C_\mu (C_1 - 1)(C_2 - 1)\langle(\partial u/\partial y)^2\rangle \varphi \tag{23}$$

with the initial condition

$$(\varphi'/\varphi)_0 = (C_2 - 1)(\varepsilon/k)_0 . \tag{24}$$

For any combination of ε and k of the form $F = k^n \varepsilon^m$, with allowance for Eqs. (20) and (21) it is possible to write

$$dF/dt = F\left\{(n + C_1 m) C_\mu \langle(\partial u/\partial y)^2\rangle (k/\varepsilon) - (n + C_2 m)(\varepsilon/k)\right\}.$$

Then, taking into account definition (22) of the function φ and Eq. (23) for it, we express an arbitrary combination of ε and k in terms of φ and φ' in the following manner:

$$F/F_0 = (\varphi'/\varphi_0')^a (\varphi/\varphi_0)^b ,$$

where $a = (n + mC_1)/(C_1 - 1)$; $b = -(n + mC_2)(C_2 - 1)$. In particular, for the turbulent viscosity ($n = 2$, $m = -1$) we have

$$\nu_t/\nu_{t0} = (\varphi'/\varphi_0')^a (\varphi/\varphi_0)^b , \tag{25}$$

where $a = (2 - C_1)(C_1 - 1)$; $b = -(2 - C_2)(C_2 - 1)$.

As is known, the solution of a linear second-order equation in general form can be represented as a linear combination of two linearly independent solutions: $\varphi = A_1\varphi_1 + A_2\varphi_2$. Since all the expressions of interest involve only the ratios of φ and φ' to their initial values, it is possible to eliminate one of the constants, for example, A_2, assuming it to be equal to 1. The second constant is determined from initial condition (24).

Since the initial value of ε is usually prescribed in the form $\varepsilon_0 = C_\varepsilon k_0^{3/2}/L$, we obtain

$$A_1 = (C\varphi_{20} k_0^{1/2} - \varphi_{20}')/(\varphi_{10}' - C\varphi_{10} k_0^{1/2}) , \tag{26}$$

where $c = C_\varepsilon/(C_2 - 1)L$.

Let us assume that the dependence of φ on the initial level of turbulence (i.e., on k_0) is completely concentrated in the coefficient A_1, i.e., the solution of Eq. (23), depending on the problem parameters, looks like

$$\varphi = A_1 (k_0, \Delta u_0) \varphi_1 (\Delta u_0) + \varphi_2 (\Delta u_0) , \tag{27}$$

which seems to be valid at least if the initial turbulence intensity is small compared to that generated in the mixing layer. Then, substituting Eqs. (26) and (27) into (25), we come to the following expression for the turbulent viscosity:

$$\nu_{t.ef}/\nu_{t0} = [f_1 + f_2/k_0^{1/2}]^a [f_3 + f_4 k_0^{1/2}]^b , \tag{28}$$

where the functions f_i depend on the initial difference in the flow velocities Δu_0, but not on the initial turbulence k_0.

Taking the values of the empirical coefficients $C_1 = 1.44$, $C_2 = 1.92$, typical for the $k-\varepsilon$ model, we obtain -0.08 in the exponent of the second factor and we will consider this factor to be constant and equal to unity. Then we represent the functions f_1 and f_2 in the form of a series in powers $\Delta u_0/u_\infty$ and limit ourselves to quadratic terms. Approximation by results of numerical calculations gave

$$f_1 = 1 - 2.8\,\Delta u_0/u_\infty + 6.0\,(\Delta u_0/u_\infty)^2 ,$$

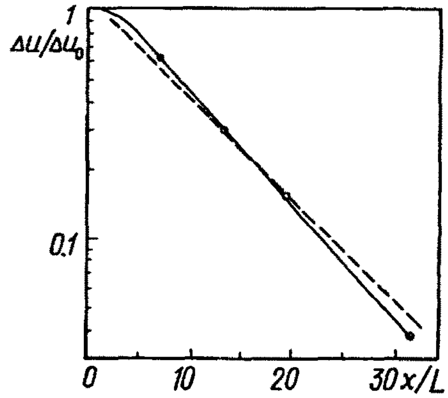

Fig. 3. Change in the velocity defect in turbulent mixing: points) experiment [4]; solid curve) numerical calculation; dashed curve) formulas (19), (28), (29).

$$f_2 = (-0.031\,\Delta u_0/u_\infty - 0.18\,(\Delta u_0/u_\infty)^2)\,\Delta u_0\,. \tag{29}$$

The error in determining the exponent in expression (19) using Eq. (28) and approximation (29) does not exceed 10% for values of $\Delta u_0/u_\infty$ from 0 to 0.67 and $k_0^{1/2}/\Delta u_0$ in the interval 0.15–0.3.

The relations obtained for turbulent mixing (19) need experimental verification. Figure 3 presents experimental data, obtained in [4], on the change in the velocity defect over the main portion of mixing in a system of plane isothermal turbulent jets. Unfortunately, experimental data on the initial turbulence level or the magnitude of the turbulent viscosity over the main portion of mixing are not presented. For comparison, we give results of a numerical calculation for $k_0^{1/2}/u_\infty = 0.0117$ (the solid curve). As is seen from this figure, in both the calculation and the experiment the velocity defect over the main portion of mixing decreases according to an exponential law with an exponent of ~ 0.112. Results of calculations carried out using formulas (19), (28), and (29) are illustrated by the dashed curve. The value of the exponent calculated from these formulas is 0.102. Other existing experimental results, including ones for nonisothermal jets [3-6], also obey an exponential relationship.

Thus, mixing in a system of cocurrent jets over the main portion obeys the exponential law (11) and (19).

NOTATION

x, y, coordinates; u, v, longitudinal and transverse velocity components; ρ, gas density; P, pressure; L, distance between the axes of adjacent jets; l_1, l_2, half-heights of the flows; ν, gas viscosity; Re, Reynolds number; λ, characteristic length of jet mixing; a_j, coefficients in the Fourier-series expansion for the longitudinal velocity; Δu, velocity defect; Y_k, concentration of the k-th component; Sc, Schmidt number; k, energy of turbulent pulsations; ε, rate of turbulent-energy dissipation; φ, function in Eq. (22). Subscripts: 0, values in the initial cross section; ∞, characteristics of the completely mixed flow; overbar, dimensionless quantities.

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